# Micromechanical aspects of granular ratcheting


Fernando Alonso-Marroquín*, Sergio-Andres Galindo-Torres* and Yucang Wang†

*MoSCoS, School of Physical and Mathematical Sciences, the University of Queensland, St. Lucia, Qld. 4067 Australia

† ESSCC, The University of Queensland, St. Lucia, Qld, 4067 Australia



**Abstract.**

The existence of granular ratcheting as a long-time behavior in granular materials is still under discussion in the scientific and engineering community. This behavior refers to the constant accumulation of permanent deformation per cycle, when the granular sample is subjected to loading-unloading stress cycles with amplitudes well bellow the yield limit. Ratcheting regimes are observed in both numerical and physical experiments. There is no controversy about the existence of ratcheting when the stress amplitudes reach the yield criterion. However, it is not clear whether this effect persists for loading amplitudes well bellow the yield limit, or whether there is a certain regime where no accumulation of deformation occurs. Early numerical simulations suggested that ratcheting may persist for extremely small loading amplitudes (Alonso-Marroquin and H. J. Herrmann, Rev. Lett. 92 5 (2004) 054301). More recent investigations drive to the conclusion that ratcheting at low stress amplitudes may be strongly influenced by the selection of the contact force (McNamara et al. Phys. Rev. E 77 (3) (2008) 31304). Here we present a numerical investigation of the dependence of granular ratcheting on contact force in a packing of spheres using the Cundall-Strack model and the McNamara correction to the contact force. We conclude that ratcheting for spherical particles is strongly infuenced by the McNamara correction. The question of the existency of genuine ratcheting for small cycles for non-spherical particles is still unsolved.



**Keywords:** granular materials, cyclic loading

**PACS:** 81.05.Rm


## 1. INTRODUCTION

Ratcheting is an interesting topic in both Physics, Material Science and Civil Engineering. The original interest of studying ratcheting comes from the possibility to extract work from noise [1, 2, 3]. Brownian motors, quantum ratchets or molecular pumps, all these machines operates under a similar ratcheting mechanism: The chaotic Brownian motion of the microworld cannot be beaten, but one can take advantage of it [3]. Many man-made ratchet devices have been constructed, and they are used as mechanical and electrical rectifiers [3]. Apart from these fascinating machines, the ratchet effect is used to describe economical or sociological processes where the intrinsic asymmetry in the system allows rectification of an unbiased input [4]. In geological materials, ratcheting is a major cause of deterioration when the material is subjected to cyclic loading, thermal or mechanical fluctuations [5, 6, 7]. An asymmetry in a foundation can produce tilting and eventual collapse of an engineering structure due to ratcheting [8]. The tower of Pisa [9] is a well documented structure, where the tilt has been observed from its construction in 1174. Railway design is another important example. Granular materials are used as a supportive railbed. The excitations caused by trains induces permanent deformation in the granular bed [10]. Therefore a better understanding of the ratcheting response will reduce the maintenance cost of many engineering structures.

The best motivated example of a ratcheting device is presented in Chapter 46 of the Feynman Lectures on Physics [11]. As shown in Fig. 1, the ratchet consist of a pawl that engages the sloping teeth of a wheel, permitting motion in one direction only. In Feynman's ratchet, an axle connects this wheel with some vanes, which are surrounded by a gas. The vanes are randomly hit by the gas molecules, but due to the presence of the pawl, only

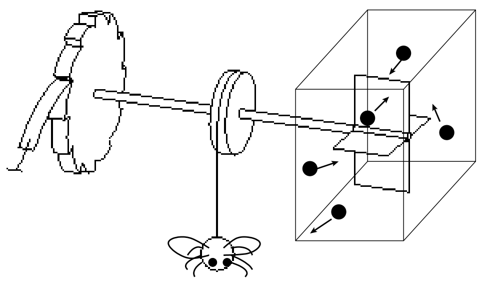

**FIGURE 1.** Microscopic ratchet device introduced by Feynman [11]. A wheel that can only turn one way is connected to a vane by an axle. The vane is inside a box with gas molecules in thermal equilibrium. Molecules randomly hit the vane. The sloping teeth of the wheel rectify the motion. The whole device converts random motion in work that can be used to lift a fly.

collisions in one direction can make the wheel lift the pawl and advance it to the next notch. An similar mechanism has been suggested for the accumulation of permanent deformation of granular materials in cyclic loading, The origin of the effect is a ratchet like deformation at the contacts resulting from an intrinsic asymmetry in the contact force [12, 13, 14].

The existence of ratcheting as a long-time behavior in granular materials is still a controversial question. Granular ratcheting refers to the constant accumulation of permanent deformation per cycle, when the granular sample is subjected to loading-unloading stress cycles with amplitudes well bellow the yield limit [15]. There are experiments on cyclic loading showing that the permanent deformation under cyclic loading accumulates beyond $10^5$ cycles [16, 5]. This appears to contradict the classical assumption of the *shakedown theory*, that the permanent deformation stops after some cycles [17].

Ratcheting regimes are observed numerical experiments [18, 19, 20, 21, 22]. This regime may persist for loading amplitudes bellow the yield limit and quasistatic deformations [18, 22]. Different works reported on similar results with the recently reported ratcheting regime in packings of disks [22] and spheres [20] indicates that this effect does not depend on the geometry of the grains, and that it may be inherent to the particle interactions.

Using numerical simulations of packing of disks and a rigorous geometrical analysis, McNamara et al showed that ratcheting in these systems is inherent from the classical Cundall-Strack frictional force [23]. They proved that this force leads in some cases to path dependency in the potential energy, even when sliding is hindered in the simulations. They also showed that alternative methods for calculating tangential forces in packing of disks remove granular ratcheting. This work open the question of the physical correctness of the contact force model used in most existing codes for capturing the frictional nature of granular systems.

Until now the NcNamara approach applies only for circular and spherical particles, so that the physical correctness of ratcheting in non-spherical particles is still not clear. The first numerical observation of ratcheting regimes were observed using polygonal packings [18]. Modeling interactions between polygons still poses serious limitations, because of the difficulty to derive conservative elastic forces: When forces between polygons are calculated as a function of their overlapping area, the energy conservation is not guaranteed [24]. An alternative approach is to define the potential energy as a function of overlap, and derive from this potential contact forces and torques [24]. However, this approach leads to unrealistic interactions, because the magnitudes of the torque applied to each particle are the same. Moreover, the derivation of forces from this potential leads to complicated expressions which are difficult to code and to extend to 3D. A simpler approach has been proposed, where a potential energy is associated to each vertex-edge interaction between the polygons [14], however, the derivation of the McNamara correction of the frictional force is still unsolved.

## 2. DEPENDENCY ON CONTACT FORCES

We investigate how the cyclic loading response of granular materials depends on the selection of contact force. We use biaxial test simulation of 288 spherical particles. The sample is confided by six walls. The front, back and bottom walls are kept fixed (Fig 2). Initially the sample is compacted by applying a constant pressure in the three moving walls while the friction coefficient is set to zero in order to achieve maximum packing. Then the friction coefficient is set to $\mu = 0.4$ while the pressure is kept constant. When the sample relaxes, the stress a the top wall is varied cyclically;

$$\begin{aligned} \sigma_{xx} &= p_0 \\ \sigma_{zz} &= p_0(1 - \Delta\sigma\cos(2\pi t/T)) \end{aligned} \tag{1}$$

The cyclic load behavior is characterized by hysteresis in the stress-strain response. This hysteresis result from the inherent hypoplastic behavior of granular materials, which means that any load involves sliding contacts, and therefore irreversible deformations. For the first cycles the hysteresis loops move, given place to accumulation of permanent deformation. In the limit of large deformation the stress seems to reduce, and the big question is whether the permanent deformation stop (*shakedown*) or if it accumulated in a small rate (*ratcheting*).

We use a visco-elastic model, were the interaction are calculated from the overlap between the spheres. The force between two overlapping spheres is separated as

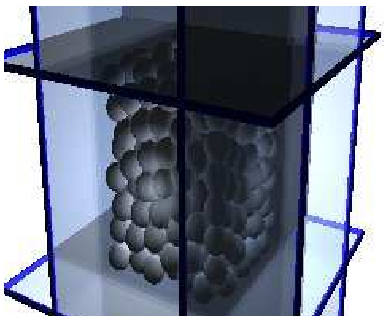

**FIGURE 2.** The experimental setup with 288 spheres. The front, back and bottom walls are kept fixed while confining pressure is applied to the other three walls.

$\vec{F}_{ij} = \vec{F}^e + \vec{F}^v$ where $\vec{F}^e$ and $\vec{F}^v$ are the elastic and viscous contribution. The viscous force is proportional to the contact velocity. The elastic part of the contact force is decomposed as $\vec{F}^e = \vec{F}^e_n + \vec{F}^e_t$, The normal force is given by $\vec{F}_n = k_n D_n \hat{n}$, where the unit normal vector $\hat{n}$ goes in the direction connecting the center of mass of the two disk. $D_n$ is the overlapping length between the spheres. $k_n$ is the normal stiffness.

The frictional force is calculated as:

$$\vec{F}^e_t = -k_t \vec{D}_t, \tag{2}$$

where $k_t$ is the tangential stiffness. The elastic displacement $D_t$ is calculated as the time integral of the tangential velocity at the contact $\dot{\vec{D}}_t$ during the time where the elastic condition $|\vec{F}^e_t| < \mu |\vec{F}^e_n|$ is satisfied. The sliding condition is imposed when $|F^e_t| > \mu F^e_n$. In this case $D_t$ is calculated from the equation $|F^e_t| = \mu F^e_n$.

The tangential velocity at the contact is calculated as

$$\dot{\vec{D}}_t = \vec{V} - (\vec{V} \cdot \hat{n})\hat{n} \tag{3}$$

Where $\vec{V}$ is the velocity at the contact

$$\vec{V} = \vec{\omega}_i \times \vec{r}_i - \vec{\omega}_j \times \vec{r}_j + \alpha(\vec{v}_i - \vec{v}_j)\hat{t} \tag{4}$$

and $\alpha$ as a correction of the classical Cundall-Strack model [23]

$$\alpha = \frac{r_i - r_j}{|\vec{X}_i - \vec{X}_j|} \tag{5}$$

In the Cundall model $\alpha = 1$, which is a good approximation in case that the particles are rigid enough so the overlap between them can be neglected. In practice, however, one has to deal with soft particles for the sake of computer time, so that $\alpha$ may become lower than one.

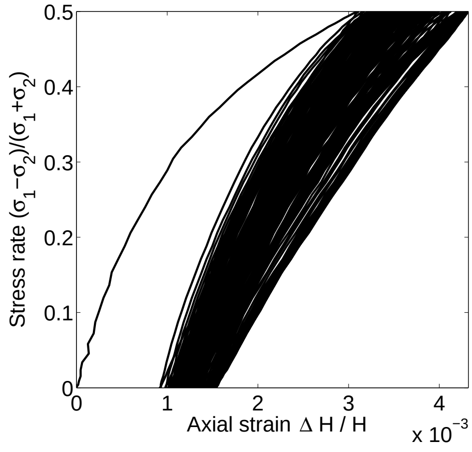


**FIGURE 3.** Stress ratio for $\alpha = 1$

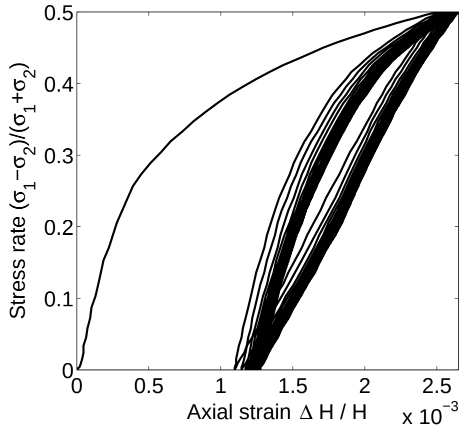


**FIGURE 4.** Stress ratio for $\alpha$ given by Eq. (5)

## 3. RESULTS

We show the results of the measured stress ratio vs axial strain for both $\alpha = 1$ and the value given by Eq. (5). The results are shown in Fig. 3 and 4. As can be seen for the case of $\alpha = 1$ there is a slow but continuous accumulation of strain with each passing cycle. However this disappear when the McNamara's correction is applied and the system enters a state of constant limit strain.

These features can be appreciated more clearly if we plot the axial strain for each cycle. In Fig. 5 and 6 the suspected behavior is confirmed. Indeed there is an accumulation of strain with $\alpha = 1$ that disappear with the correction. So far, the ratcheting produced by the Cundall model is due to a lost in elastic potential energy when the contact is in the non-sliding regime.

However, ratcheting can still be produced by the new friction model. Moreover although the model introduce the conservation of the tangential force, it is not applicable for shapes different to spheres. For example, in simulation based on spheropolygons [14] the Eq. (5) is not valid and in most cases the tangential force in the non-sliding regime is conservative.

## 4. CONCLUDING REMARKS

Ratcheting in spherical particles strongly depends on the contact interaction. In particular, the selection of the Cundall-Strack contact force model leads to ratcheting, while the correction proposed by McNamara removes ratcheting. This is, however, not a prove of the non-existence of ratcheting. Ther question of the validity of ratcheting with non-spherical particles is still open. In particular, the model of polygonal particles [18] of spheropolygons [14] the McNamara correction is not

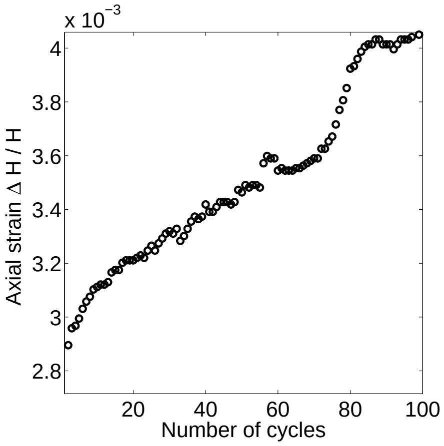


**FIGURE 5.** Strain ratio for $\alpha = 1$

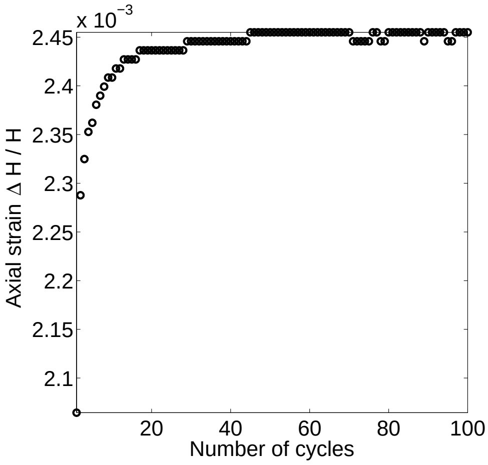


**FIGURE 6.** Strain ratio for $\alpha$ given by Eq. (5)

valid, because it depends on the curvature of the particles.

There is also a strong necessity to understand why realistic soils like railroad bed and pavement accumulates permanent deformation. There is al effect of wearing, or a combined effect of wearing and ratcheting? the question is unsolved.

## ACKNOWLEDGMENTS

We acknowledge the support of the University of Queensland Early Carrier Research Grant and the Australian Research Council (Discovery project No.DP0772409).

## REFERENCES

1. I. Zapata, R. Bartussek, F. Sols, and P. Hänggi, *Phys. Rev. Lett.* **77**, 2293 (1996).
2. J. Howard, *Nature* **389**, 561 (1997).
3. P. Reimann, *Phys. Rep.* **361**, 57 (2002).
4. P. H. Dybvig, *The Review of Economic Studies* **62**, 287–313 (1995).
5. F. Lekarp, A. Dawson, and U. Isacsson, *J. Transp. Engrg.* **126**, 76–82 (2000).
6. G. Royer-Carfagni, "Granular decohesion thermal damage in marble monuments," in *Novel approaches in civil engineering*, Springer-Verlag, Berlin, 2004, pp. 177–185.
7. M. Huang, Z. Suo, Q. Ma, and H. Fujimoto, *J. Mater. Res.* **15**, 1239–1242 (2000).
8. G. L. England, T. D. C. M. Tsang, N. Mihajlovic, and J. B. Bazaz, "Ratcheting flow of granular materials," in *Static and Dynamic Properties of Gravelly Soils*, edited by M. D. Evans, and R. J. Fragaszy, ASCE, 1995, pp. 64–76.
9. J. Burland, and C. Viggiani, *Rivista Italiana di Geotecnica* **28**, 179–200 (1994).
10. S. Lobo-Guerrero, and L. E. Vallejo, *Granular Matter* **8**, 195–204 (2006).
11. R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison - Wesley, Massachusetts, 1963, vol. 1, chap. 46, pp. 46.1–46.9.
12. G. Gudehus, *J. Stat. Mech.* **P07022** (2006).
13. G. Gudehus, *Physical Soil Mechanics*, Springer, Berlin, 2008, chap. 4.9, iSBN: 978-3-540-36353-8.
14. F. Alonso-Marroquin, *Europhysics Letters* **83**, 14001 (2008).
15. G. Gudehus, *Granular Matter* **8**, 93–102 (2006).
16. T. Triantafyllidis, editor, *Cyclic Behaviour of Soils and Liquefaction Phenomena*, A.A. Balkema Publishers, Leiden, 2004.
17. R. W. Sharp, and J. R. Booker, *Journal of Transportation Engineering* **110**, 1–14 (1984).
18. F. Alonso-Marroquin, and H. J. Herrmann, *Phys. Rev. Lett.* **92**, 054301 (2004).
19. R. Garcia-Rojo, and H. J. Herrmann, *Granular Matter* **7**, 109–118 (2005).
20. C. T. David, R. García-Rojo, H. J. Herrmann, and S. Luding, "Hysteresis and creep in powders and grains," in *Proceedings of Powders and Grains 2005*, edited by R. Garcia-Rojo, H. Herrmann, and S. McNamara, Balkema, Leiden, 2005, pp. 291–294.
21. A. A. Peña, A. Lizcano, F. Alonso-Marroquin, and H. J. Herrmann, "Investigation of the asymptotic states of granular materials using a discrete model of anisotropic particles," in *Proceedings of Powders and Grains 2005*, Balkema, 2005, vol. 1, pp. 697–700.
22. R. García-Rojo, F. Alonso-Marroquin, and H. J. Herrmann, *Phys. Revi. E* **72**, 041302 (2005).
23. S. McNamara, R. García-Rojo, and H. Herrmann, *Physical Review E* **77**, 31304 (2008).
24. T. Poeschel, and T. Schwager, *Computational Granular Dynamics*, Springer, Berlin, 2004.